Third Semester B.E. Degree Examination, June-July 2009
Signals and Systems

Max. Marks:100

Note: *Answer any FIVE full questions.*

1 a. The trapezoidal pulse x(t) shown in figure Q1 (a) is defined by,

$$x(t)=\begin{cases} 5-t & 4\le t\le 5 \\ 1 & -4\le t\le -4 \\ t-5 & -5\le t\le -4 \\ 0 & \text{otherwise} \end{cases}$$

Determine the total energy of x(t), **(05 Marks)**

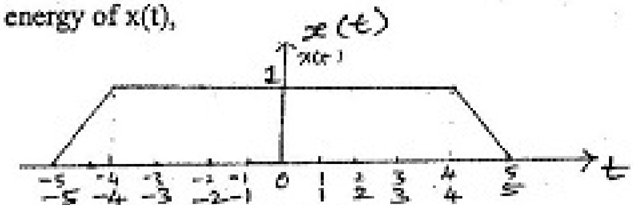

Fig. Q1 (a)

b. Given the signal x(t) and y(t) in figure Q1(b) and figure Q1 (c) respectively. Carefully sketch the following signals. i) x(t)y(t-1) ii) x(t+1)y(t-2). **(05 Marks)**

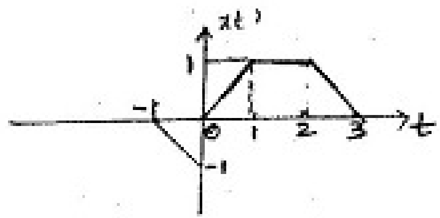

Fig. Q1 (b)

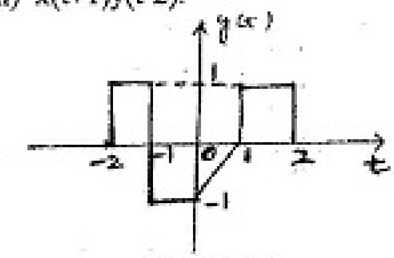

Fig. Q1 (c)

c. Sketch the waveform of the signal given below,
$y(t)=r(t+2)-r(t+1)-r(t-1)+r(t-2)$ **(05 Marks)**

d. Determine whether the following signals are periodic. If they are periodic, find the fundamental period.
i) $x(n)=\cos\left(\frac{8}{15}\pi n\right)$ ii) $x(t)=v(t)+v(-t)$ where $v(t)=\cos(t).u(t)$ **(05 Marks)**

2 a. Determine whether each of the systems given below is linear, time invariant, causal and memory.
i) $y(t)=\cos(x(t))$ ii) $y(n)=2x(n)u(n)$ iii) $y(t)=\frac{d}{dt}\left\{e^{-t}x(t)\right\}$ **(12 Marks)**

b. Sketch the trapezoidal pulse y(t) that is related to figure Q1 (a) as follows:
$y(t)=x(10t-5)$ **(08 Marks)**

3 a. Use the definition of the convolution sum to prove the following properties:
i) $x(n)*(h(n)+g(n))=x(n)*h(n)+x(n)*g(n)$
ii) $x(n)*(h(n)*g(n))=(x(n)*h(n))*g(n)$
iii) $x(n)*h(n)=h(n)*x(n)$ **(12 Marks)**

b. Evaluate continuous time convolution integral given below:
$y(t)=(t(u(t)+(10-2t)u(t-5)-(10-t)u(t-10)*u(t))$ **(08 Marks)**

4 a. Draw direct form I and direct form II implementation of the system given below:

$$\frac{d^3y(t)}{dt^3}+2\frac{dy(t)}{dt}+3y(t)=x(t)+3\frac{dx(t)}{dt}$$ **(06 Marks)**

4 b. Find the expression for the impulse response the input x(t) to the output y(t) in terms of impulse response for the LTI system shown below figure Q4 (a). **(06 Marks)**

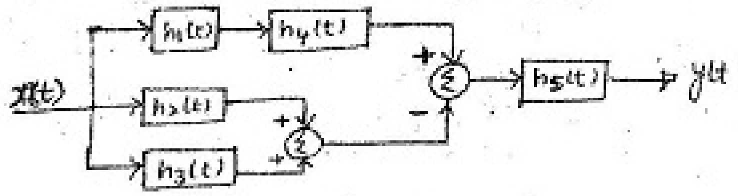

Fig. Q4 (a)

c. Determine the natural response for the system described by the following differential equation,

$$\frac{d^2y(t)}{dt^2}+5\frac{dy(t)}{dt}+6y(t)=2x(t)+\frac{dx(t)}{dt}, \quad y(0)=2, \quad \left.\frac{dy(t)}{dt}\right/_{t=0}=1$$ **(08 Marks)**

5 a. Prove the following properties related to DTFS:
i) Frequency shift property. ii) Convolutional property. iii) Modulation property. **(14 Marks)**

b. Use the definition of the DTFS to determine the time signals represented by the following DTFS coefficients:
$X(K)=\cos\left(\frac{6\pi}{17}K\right)$ **(06 Marks)**

6 a. Determine the signal x(n) if its DTFT is as shown in figure Q6 (a) and Q6 (b). **(12 Marks)**

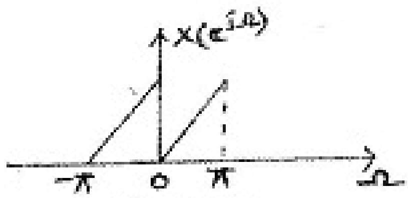

Fig. Q6 (a)

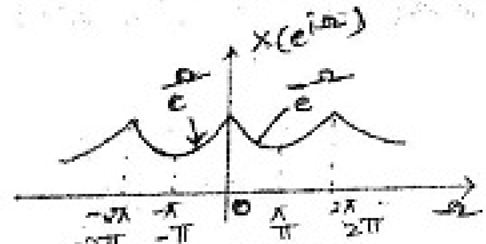

Fig. Q6 (b)

b. Prove that, $x(n) \xleftrightarrow{\text{DTFT}} x(e^{j\Omega})$ then $\sum_{n=-\infty}^{\infty}|x(n)|^2=\frac{1}{2\pi}\int_{2\pi}|x(e^{j\Omega})|^2\,d\Omega$. **(08 Marks)**

7 a. State and prove final value theorem of Z-T. **(05 Marks)**

b. Prove the differential property of the Z-T. **(05 Marks)**

c. Find Z-T of $x(n)=\left\{\left(\frac{1}{2}\right)^n+\left(\frac{1}{3}\right)^n\right\}u(n)$. Plot the ROC units pole zero diagram. **(05 Marks)**

d. Determine the z-transform of the signal given using z-transform property,
$x(n)=\left(\frac{1}{2}\right)^n u(n) * \left(\frac{1}{3}\right)^n u(n)$ **(05 Marks)**

8 a. Find the inverse Z-T of $x(z)=\frac{z}{3z^2-4z+1}$, for the ROC as i) $|z|>1$ ii) $|z|<\frac{1}{3}$
iii) $\frac{1}{3}<|z|<1$
using partial fraction expansion method. **(10 Marks)**

b. Use a power series expansion to determine the time domain signal corresponding to the following z-transform $x(z)=\cos(2z)$, $|z|<\infty$. **(05 Marks)**

c. Determine x(n) for $x(z)=e^z+e^{1/z}$. **(05 Marks)**

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